# Two Approximate Approaches for Reliability Evaluation in Large Networks. Comparative Study

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***Abstract* — This paper deals with the complex problem of reliability evaluation of stochastic networks in which both links and nodes failures are considered, and compares two approximate approaches able to reduce the computation time: a sum of disjoint products (SDP) approach and another based on Monte Carlo simulation. In case of the SDP approach, the reliability is computed based on the minimal paths. In a first stage, only the links of the network are considered, and a 'multiple variables inversion' technique for developing the set of minimal paths into a sum of disjoint products is applied. Then, in a second stage, each term of the set of disjoint link-products is processed separately taking into consideration the reliability values for both links and adjacent nodes. In case of Monte Carlo simulation, for speeding up the method and reducing the computation time, both minimal paths and cuts are considered. Also, other acceleration techniques are applied.**

***Keywords— two-terminal network reliability; link and node failure; SDP algorithm; minimal paths; multiple variable inversion technique; Monte Carlo simulation***

## I. Introduction

The theory of network reliability is applied in the real-world systems that can be modeled as stochastic networks, such as communications systems, networks of sensors, social networks etc. Generally speaking, reliability or availability indices describe the ability of a network to perform a desired operation. Mostly, the study is limited to the operation between two given nodes (two-terminal reliability evaluation). Starting from a structure function expressed in terms of minimal paths or cuts ([1]-[4]), a SDP technique is then used to determine an equivalent function in the form of a sum of disjoint products [5]-[8]. Unfortunately, the problem of computing the network reliability based on SDP algorithms is NP-hard [6]. The problem of network reliability evaluation is even more complicated when both links and nodes failures are considered [9], [10]. Because of this, in case of large networks, other techniques for performance evaluation are also applied, such as those based on network decomposition ([11]-[14]) or Monte Carlo simulation [15]-[20].

To reduce calculation time, in most such works the authors assume that the nodes of the network are perfectly reliable. The basic idea for this simplified approach is that the failure of a node inhibits the work of all links connected to it, so that starting from the given network with unreliable nodes, reduced models with perfect nodes but with links having increased failure probabilities can be obtained. This approach is simpler but not very precise. As the failure of a node obstructs the work of all adjacent links, the work of these links depends on the state of this common node. However, the model is solved under the assumption that the failures that may occur in the network are independent. For this reason, the reliability estimation should be accepted with caution. Indeed, the error of reliability estimation is unacceptable in many cases, especially when the probabilities of node failure have higher values. For this reason, we do not use such simplified models in this work.

In this paper we are focused on the problem of approximate evaluation of reliability or availability in large networks, when both links and nodes failures are considered, and in this regard, an approximate SDP algorithm and a Monte Carlo simulation technique are compared.

The approximate SDP algorithm for two-terminal network reliability evaluation we consider involves two stages. In the first stage, the method is focused only on the links of the network. For the two given nodes, all the minimal paths are enumerated, and then, this set of minimal paths is transformed into a set of disjoint products. In the second stage, each term of the sum of disjoint products including only link state variables (i.e., link-product) is processed distinctly by considering the reliability values of both links and adjacent nodes. In this way, a good accuracy is obtained.

The rest of this paper is organized as follows. Section II introduces notations, a nomenclature and some preliminary considerations, while section III presents general issues regarding the network reliability evaluation. Section IV provides a method for exact evaluation of two-terminal network reliability when both nodes and links failures are considered. Section V presents an approximate SDP approach that reduces the complexity of this problem in medium-to-large networks, while section VI describes a Monte Carlo simulation technique. Section VII presents comparative results and finally, in Section VIII, some conclusions are drawn regarding this work.

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## II. PRELIMINARIES

### A. Nomenclature

a) *Reliability*. The two-terminal reliability of a stochastic network expresses the probability that at least one path between the two given nodes operates successfully.

b) *Minimal path.* A minimum set of links and their adjacent nodes whose good operation is sufficient to ensure the connection between two given nodes.

c) *Minimal cut.* A minimum set of links and/or nodes whose failure disconnects two given nodes.

d) *Uniproduct.* A Boolean product composed of different uncomplemented variables.

e) *Mixproduct.* A product of one uncomplemented subproduct and one or more complemented subproducts.

f) *Disjoint products.* A set of products expressing mutually exclusive conditions.

### B. Notations

a) $G(V, E)$ is a network model with node set $V = \{ y_1, y_2, \cdots, y_k\}$ and link set $E = \{ x_1, x_2, \cdots, x_m\}$ ;

b) $s, t \in V, s \neq t$ , are the source and target nodes;

c) $p_x$ is the reliability of node $x \in V$ or link $x \in E$ , and $q_x = 1 - p_x$ ;

d) $R_{s-t}$ is the reliability of the network model with *s* and *t* the two given nodes ( $s-t$ network reliability).

e) $Pr(A)$ is the probability of the event *A*.

### C. Assumptions

For this study, we assume that a network component (i.e., node or link) is either operational or failed, so a logical variable is used to denote its state. Also, we assume that all failures in the network are independent.

## III. CONSIDERATIONS ON NETWORK RELIABILITY EVALUATION

For the network model $G(V, E)$ and two given nodes, let the minimal path set be $MPS = \{MP_1, MP_2, \cdots, MP_{np}\}$ . A minimal path $MP_i \in MPS$ is described by a product of logical variables associated with some network elements, and the reliability of this path is

$$Pr(MP_i) = \prod_{x \in MP_i} p_x \ . \tag{1}$$

Based on *MPS*, the structure function $S = \bigcup_{i=1}^{np} MP_i$ is defined, and the reliability $R_{s-t}$ is expressed with the relation

$$R_{s-t} = Pr(S) = Pr\left( \bigcup_{i=1}^{np} MP_i \right). \tag{2}$$

To compute $R_{s-t}$ based on (2), the well-known rule of SDP can be applied [6]:

$$Pr\left( \bigcup_{i=1}^{np} MP_i \right) = Pr(MP_1) + Pr(\overline{MP_1} \cap MP_2) + Pr(\overline{MP_1} \cap \overline{MP_2} \cap MP_3) + \cdots + Pr(\overline{MP_1} \cap \overline{MP_2} \cap \cdots \cap \overline{MP_{np-1}} \cap MP_{np}) \ . \tag{3}$$

Consequently, *S* is developed in an equivalent function $S'$ comprising only disjoint products (*DP*), so that the reliability $R_{s-t}$ can be obtained with the relation

$$R_{s-t} = Pr(S') = Pr\left( \bigcup_j DP_j \right) = \sum_j Pr(DP_j) \ . \tag{4}$$

In other words, based on the set of minimal paths, the evaluation of network reliability is essentially reduced to the problem of generating an equivalent set of disjoint products.

An SDP method based on ‘single variable inversion’ (SVI) for network reliability assessment has been presented for the first time by Aggarwal, Misra, and Gupta [21]. Subsequently, other better SVI methods were reported. But, a more effective approach is based on ‘multiple variable inversion’ (MVI) techniques when a product may contain one or more complemented subproducts. An excellent survey on SVI and MVI techniques can be found in [5], [7], or [8]. A new MVI method, called NMVI, is proposed by Caşcaval and Floria in [1]. As shown in [1], NMVI is an efficient method, providing fewer disjoint products compared with other well-known MVI techniques. This is the SDP algorithm we consider in this work.

In the next sections we discuss the problem of network performance evaluation, in which both links and nodes failures are considered. First, an exact method of reliability evaluation is given. Then, the approximate methods we compare in this work are presented.

## IV. EXACT EVALUATION OF NETWORK RELIABILITY

For a network model an exact computation of the two-terminal reliability can be obtained by using the set of minimal paths that include both links and adjacent nodes. Compared to the case where the study is limited to the links of the network, when the nodes are also considered, the number of minimal paths is unchanged, but any term is extended by including the adjacent nodes.

To illustrate this method, let us analyze the network $N_1$ presented in Fig. 1, with 1 and 4 the source and target nodes. In the reliability model, these two terminal nodes are considered in series with the rest of the network.

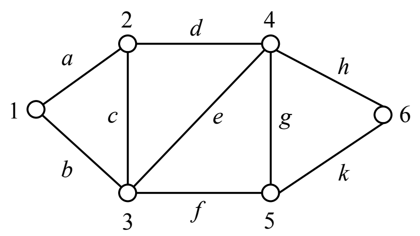



**Fig. 1 - Network model with unreliable nodes ($N_1$)**

For the two given nodes, the set of minimal paths composed of both nodes and links is

$$MPS_{1-4} = \{2ad, 3be, 23ace, 23bcd, 35bfg, 235acfg, 356bfhk, 2356acfhk\}.$$

Based on the NMVI method, the following set of disjoint products results:

$$DPS_{1\text{-}4} = \{ 2ad\,, 3be\overline{2ad}\,,\ 23a\bar{b}c\bar{d}e\,,\ 23\bar{a}bcd\bar{e}\,, 35b\bar{e}fg\overline{2d}\,, 235\bar{a}b\bar{c}d\bar{e}fg\,,\ 235a\bar{b}c\bar{d}\bar{e}fg\,, 2356\bar{a}b\bar{c}d\bar{e}f\bar{g}hk\,,\ 356b\bar{e}f\bar{g}hk\overline{2d}\,, 2356a\bar{b}c\bar{d}\bar{e}f\bar{g}hk \}.$$

Finally, by applying (4), for the two-terminal reliability $R_{1-4}$, the following equation can be written:

$$\begin{aligned} R_{1-4} = {} & p_1 p_4 \big(p_2 p_a p_d + p_3 p_b p_e (1-p_2 p_a p_d) \\ & + p_2 p_3 p_a p_c p_e (1-p_b)(1-p_d) \\ & + p_2 p_3 p_b p_c p_d (1-p_a)(1-p_e) \\ & + p_3 p_5 p_b p_f p_g (1-p_b)(1-p_2 p_d) \\ & + p_2 p_3 p_5 p_b p_d p_f p_g (1-p_a)(1-p_c)(1-p_e) \\ & + p_2 p_3 p_5 p_a p_c p_f p_g (1-p_b)(1-p_d)(1-p_e) \\ & + p_2 p_3 p_5 p_6 p_b p_d p_f p_h p_k (1-p_a)(1-p_c)(1-p_e)(1-p_g) \\ & + p_3 p_5 p_6 p_b p_f p_h p_k (1-p_e)(1-p_g)(1-p_2 p_d) \\ & + p_2 p_3 p_5 p_6 p_a p_c p_f p_h p_k (1-p_b)(1-p_d)(1-p_e)(1-p_g)\big). \end{aligned} \tag{5}$$

As verification, observe that for the links *k* and *h* and the common node 6, the series reliability rule could be applied. For this reason, all these elements are always found together.

The problem that arises in case of exact network reliability evaluation is related to the SDP algorithms. More exactly, compared with the case in which the study is limited to the links of the network, when the adjacent nodes are also considered, the number of disjoint products increases significantly for large networks [9].

For example, in case of a network model with 780 minimal paths, by applying the NMVI method on the link-products, 48696 disjoint products are obtained. When both links and nodes are considered, the number of disjoint products grows up to 105468. In other words, with respect to the number of disjoint products, the relative growth is about 117%. With a SVI technique, the growth is even greater. For this reason, other approximate methods may be preferred for large networks.

## V. Approximate SDP Method For Network Reliability Evaluation

To reduce the computation time, in the first stage, the SDP approach is limited to the links of the network. Thus, the set of minimal link-paths are extended into a set of disjoint link-products by applying the NMVI method.

Then, in a second stage, each term of the sum of disjoint products composed only of link state variables is processed distinctly by considering both links and adjacent nodes reliability values. The nodes reliability values are taken into consideration in a specific mode for each term of the set of disjoint link-products, when only the adjacent nodes of the links that compose the current product are considered. That is the basic idea for this approximate approach.

A term *DP* in the set of disjoint products *DPS* can be a mixproduct that includes one uniproduct (*UP*) and one or more complemented subproducts. As illustrated in the previous section for the network model $N_1$, the uniproduct *UP* includes a path that connects the source and target nodes. The set of all nodes adjacent to the links that compose the uniproduct *UP* is indicated by *AN*. The probability of the network state described by *UP* is given by the equation:

$$Pr(UP) = \prod_{x \in UP} p_x \prod_{y \in AN} p_y\,. \tag{6}$$

The main problem is how to evaluate with a good accuracy the probability of a network state described by complemented subproducts.

First, let us consider a complemented variable $\bar{x}$ that describes a failed state of a link which connects two nodes, let us say $y_i$ and $y_j$. The probability corresponding to this state is computed by the equation:

$$Pr(\bar{x}) = 1 - p'_x\,, \text{ where}$$

$$p'_x = \begin{cases} p_x p_{y_i} p_{y_j} & \text{if } y_i, y_j \notin AN \\ p_x p_{y_i} & \text{if } y_j \in AN, y_i \notin AN \\ p_x p_{y_j} & \text{if } y_i \in AN, y_j \notin AN \end{cases} \tag{7}$$

Now, let us consider a complemented subproduct $\overline{x_1 x_2 \cdots x_k}$ which describes the state of inoperability for a group of *k* links of the network. The probability corresponding to this state is computed by the equation

$$Pr(\overline{x_1 x_2 \cdots x_k}) = 1 - X\,, \tag{8}$$

where the product $X$ comprises the probabilities of the corresponding links and also of the adjacent nodes that do not belong to *AN*, included only once.

To illustrate this rule, let us consider again the network model $N_1$ presented in Fig. 1, with 1 and 4 the source and the target nodes. Take, for example, the mixproduct $be\overline{ad}$ composed of the uniproduct $UP = be$ and the complemented subproduct $\overline{ad}$. For the path $be$, the set of adjacent nodes is $AN = \{3\}$. Finally,

$$Pr(be\overline{ad}) = p_b p_e p_3 (1 - p_a p_d p_2).$$

## VI. Reliability Evaluation by a Monte Carlo Simulation Approach

The network topology related to the two given nodes is described in the simulation program by a binary matrix *Q* that reflects all the minimal paths between *s* and *t*, paths that include both links and nodes. For the network model presented in Fig. 1, but with the source and target nodes 1 and 6, the set of minimal paths is

$$MPS_{1-6} = \{24adh, 34beh, 35bfk, 245adgk, 235acfk, 234aceh, 345bfgh, 345begk, 234bcdh, 2345adefk, 2345acfgh, 2345acegk, 2345bcdgk\},$$

and the matrix of capability *Q* is given in Fig. 2.

Note that, in the reliability model, the terminal nodes are considered in series with the rest of the network, so that the two-terminal reliability is computed by using the equation

$$R_{s-t} = p_s p_t R_{MC}, \tag{9}$$

where $R_{MC}$ is the probability value given by simulation.

The network state after the period of time for which the reliability is evaluated, without the nodes *s* and *t* as mentioned before, is described by a binary vector *S*. The network elements are assigned to the locations in vector *S* the same as in matrix *Q*. Remember that we need to evaluate the probability that at least one path between *s* and *t* operates successfully. Thus, to check if the nodes *s* and *t* are connected in the state *S*, one can apply the procedure presented in Fig. 3.

Based on the simulation results, the network reliability can be estimated by the equation

$$R_{MC} = \frac{ns}{NT}, \tag{10}$$

where *ns* is the number of successful attempts for the connection $s-t$, and $NT$ is the total number of trials.

In order to accelerate the Monte Carlo simulation, the following measures were implemented:

a) Only operations with integers are made to determine the state of the network at a trial;

$$\begin{array}{ccccccccccccc}
2 & 3 & 4 & 5 & a & b & c & d & e & f & g & h & k
\end{array}$$
$$\begin{bmatrix}
1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\
1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1
\end{bmatrix}$$

**Fig. 2 – *Q* matrix for the network model $N_1$ (*s* = 1, *t* = 6).**

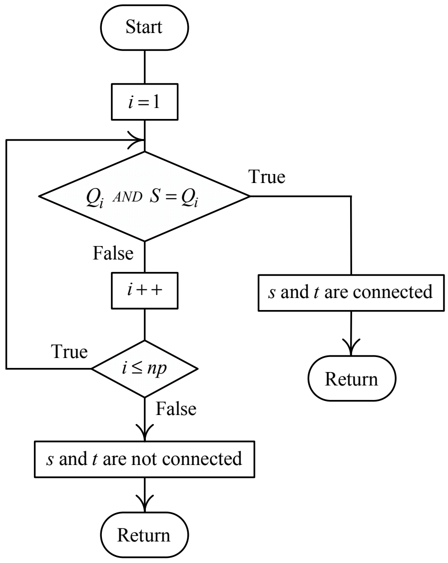



**Fig. 3 – Checking the $s-t$ connection in the network state *S*.**

b) To reduce the checking of the connection $s-t$ in the network state *S*, the paths described in matrix *Q* are ordered taking into account the corresponding probabilities calculated with (6);

c) When the network reliability is lower, the checking of the connection $s-t$ in the network state *S* is speeded up by using a reduced set of minimal cuts for the two given nodes;

d) For networks of high reliability, some paths in matrix *Q* with the highest probabilities of operation are treated distinctly. Thus, at a test, only the states for the network elements included in these paths are determined, thus avoiding many unnecessary operations.

Compared with a standard Monte Carlo method [15], by applying these acceleration measures, the simulation time is reduced to less than half. We call this method Reduced Monte Carlo (RMC) simulation.

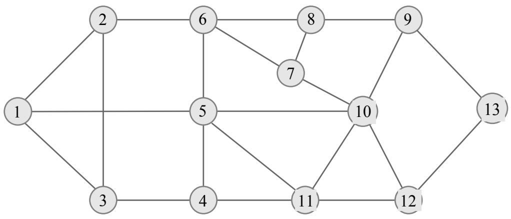
## VII. Comparative Numerical Results

To compare these two approximate approaches, the network models presented in Fig. 4 were considered. For both nodes and links, the reliability values were randomly generated in the range [0.95, 1]. The numerical results are presented in Table I. To verify the accuracy of the reliability evaluation, for the first two models, $N_2$ and $N_3$, the exact values obtained based on the SDP method presented in section IV are also included. For the RMC simulation, for a good accuracy, $10^7$ trials were generated for each test.

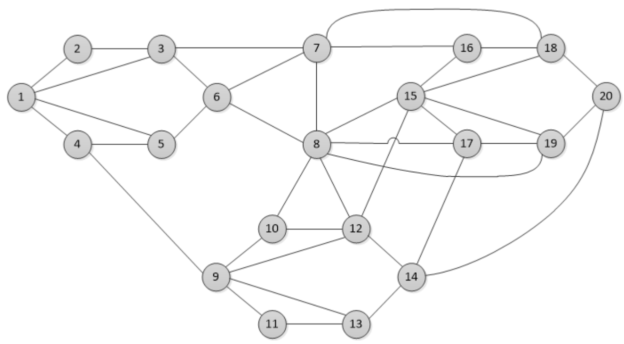
Comparative results regarding the computing time for these evaluations are presented in Table II.

With respect to the computing time for applying an SDP algorithm, for the simple network model $N_2$ with a number of hundreds of minimal paths, this value is reduced, less than one second. But, for the large network model $N_4$ with more than 70000 minimal paths, the computing time is excessively high, of the order of days.

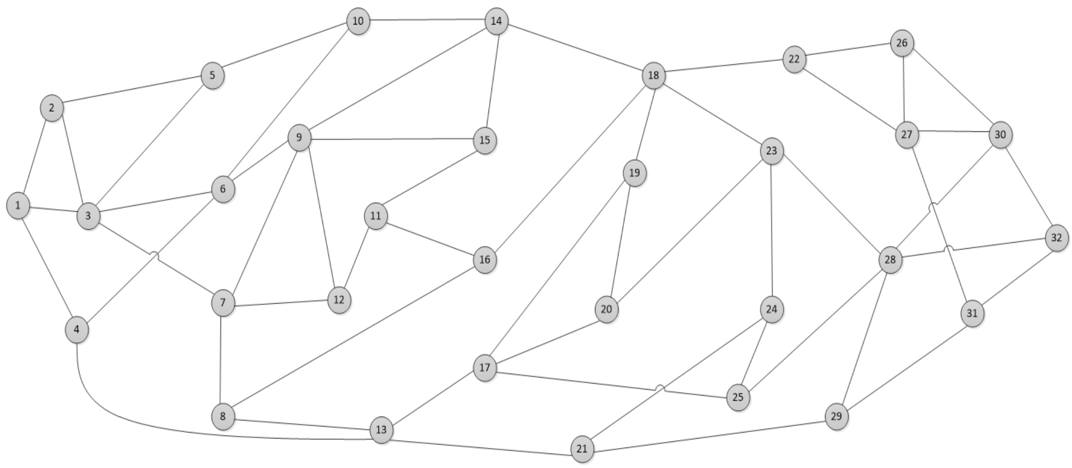
Regarding the RMC simulation, obviously, the execution time and the reliability estimation accuracy depend on the number of trials. Acceptable but not very good accuracy can be obtained with a number of trials ranging from $10^4$ to $10^5$. However, the convergence is very slow and, therefore, for high accuracy, the number of trials must be in the range $10^7$–$10^9$.




a) 13-node, 22-link network ($N_2$).




b) 20-node, 39-link network ($N_3$).




c) 32-node, 52-link network ($N_4$).

**Fig. 4 - Network models with unreliable nodes.**

Table I. Two-Terminal Network Reliability ( $R_{s-t}$ ). Comparative Results

| Network model | Connection studied | SDP approach – exact evaluation | SDP approach – approximate evaluation | RMC simulation |
|---|---|---|---|---|
| $N_2$ | $s = 1$, $t = 13$ ($np = 280$) | 0.982883 | 0.980659 | 0.983070 |
| $N_3$ | $s = 1$, $t = 20$ ($np = 11848$) | 0.978858 | 0.97670 | 0.978880 |
| $N_4$ | $s = 1$, $t = 32$ ($np = 70082$) | – | – | 0.967937 |

TABLE II. COMPUTING TIME FOR RELIABILTY EVALUATION

| Network model | SDP approach – exact evaluation | SDP approach – approximate evaluation | RMC simulation ($10^7$ trials) |
|---|---|---|---|
| $N_2$ | 0.1 s | 0.06 s | 3.3 s |
| $N_3$ | 21 min 23 s | 13 min 15 s | 13 s |
| $N_4$ | - | - | 24 min 3 s |

Having in view the reliability evaluation time, and also the estimation accuracy, we can say that for medium networks an SDP approach is recommended, whereas for large networks, an RMC simulation approach is preferable. For a very large network, the Monte Carlo simulation is the only solution.

## VIII. FINAL REMARKS

The efficiency of a method dedicated to the evaluation of network reliability depends to a great extent on the network under study. In case of complex networks, approximate approaches are necessary, especially the Monte Carlo simulation (see, for example, [17], [20]). A standard Monte Carlo simulation leads to a high computing time. For this reason, specific measures for speeding up the simulation time are strongly required. Some of them are presented in this paper.

The main disadvantage of the simulation is that the program is executed entirely for any set of reliability values associated with the nodes or the links of the network. In other words, any change in the set of reliability values for the network components, implies a new execution for the simulation program. In practice, for a connection between two given nodes, it has to set the reliability of the network components until the required reliability is reached. In this case, the Monte Carlo simulation is quite difficult to apply.

In case of a SDP approach, for a connection between two given nodes, the minimal paths and the corresponding disjoint products must be generated only once. After that, the two-terminal reliability is computed quickly based on (4), so that it is easy to adjust the reliability of the network components to reach the required connection reliability. For these reasons, the SDP methods for approximate evaluation of network reliability are necessary. The method proposed in this work is not so accurate, but it offers the advantage of a pessimistic estimation, so that the result can be treated as a lower bound. Improving the estimation accuracy by an SDP approach is a subject for future work.

For complex networks, where the simulation time is too high, other methods that lead to a network decomposition are also useful, as demonstrated in [10]-[14]. Also, an approach based on binary decision diagrams can be applied to reduce the computation time [22].

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